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Please check that this question paper contains 38 Questions and has 06 Printed pages.

D.A.V. INSTITUTIONS, CHHATTISGARH

PRACTICE PAPER 9

CLASS –XII

SUBJECT : MATHEMATICS

Time Allowed : 3 Hours **Maximum Marks : 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section - A

(Select the correct options. Each MCQ carries 1 mark)

1. If $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$, *then the value of* $(x-y)$ *is*

   a) 0 b) 10 c) -10 d) none of these.

2. If $A=\begin{bmatrix}2 & k & -3\\0 & 2 & 5\\1 & 1 & 3\end{bmatrix}$, then A^{-1} exist if

   a) $k \neq 1$ b) $k \neq 2$ c) $k \neq -2$ d) none of these

3. If the area of triangle with vertices $(1,3), (0,0)$ *and* $(k, 0)$ is 3 sq. units, then the value of k is

   a) 0 b) ± 1 c) ± 2 d) ± 3

4. If $\begin{vmatrix}x+1 & x-1\\x-3 & x+2\end{vmatrix}=\begin{vmatrix}4 & -1\\1 & 3\end{vmatrix}$, *then the value of x is*

   a)1 b) -1 c) 2 d) -2

5. The roots of the equation $\begin{vmatrix}0 & x & 16\\x & 5 & 7\\0 & 9 & x\end{vmatrix}=0$ is

   a) 0, 12 and 12 b) 0 and ±12 c) 0, 12 and 16 d) 0, 9 and 16

6. The function $f(x)=\begin{cases}\frac{\sin x}{x}+\cos x, & if\ x\neq 0\\ k, & if\ x=0\end{cases}$ is continuous at $x=0$, then the value of k is

   a) 3 b) 2 c) 1 d) 1.5

7. The derivatives of $\cos^{-1}(2x^2-1)$ w. r . t $\cos^{-1}x$ is

   a) 2 b) $\frac{-1}{2\sqrt{1-x^2}}$ c) $\frac{2}{x}$ d) $1-x^2$.

8. $\int \frac{dx}{\sin^2 x\cos^2 x}$ is equal to

   a) $\tan x+\cot x+C$ b) $(\tan x+\cot x)^2+C$

   c) $\tan x-\cot x+C$ d) $(\tan x-\cot x)^2+C$

9. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$ respectively, are

a) 2 and 4 b) 2 and 2 c) 2 and 3 d) 2 and not defined

10. The integrating factor of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is

a) $\cos x$ b) $\tan x$ c) $\sec x$ d) $\sin x$

11. If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then the value of $\vec{a}.\vec{b}$ is

a) $6\sqrt{3}$ b) $8\sqrt{3}$ c) $12\sqrt{3}$ d) none of these

12. If the projection of $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ on $\vec{b} = 2\hat{\imath} + \lambda\hat{k}$ is zero, then the value of λ is

a) 0 b) 1 c) $-\frac{2}{3}$ d) $-\frac{3}{2}$

13. The vector in the direction of the vector $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has magnitude 9 is

a) $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ b) $\frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3}$ c) $3(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ d) $9(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

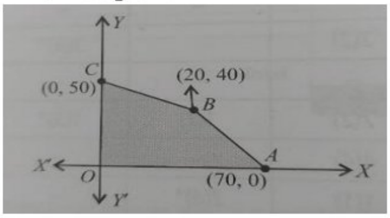
14. P is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of P is 5, then its y co-ordinate is

a) 2 b) 1 c) –1 d) –2

15. A line makes equal angles with coordinate axes. Direction cosines of this line are

a) $\pm(1,1,1)$ b) $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ c) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ d) $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

16. The feasible region OABC for a LPP is shown in the following figure. The objective function Z = 20x + 35y, will be maximum at point



a) (70,0) b) (20, 40) c) (0, 50) d) (0,0)

17. Corner points of the feasible region for an LPP are $(0,2), (3,0), (6,0), (6,8)$ and $(0,5)$. Let $F = 4x + 6y$ be the objective function. The minimum value of F occurs at

a) $(0,2)$ only

b) $(3,0)$ only

c) the mid-point of the line segment joining the points $(0,2)$ and $(3,0)$

d) any point of the line segment joining the points $(0,2)$ and $(3,0)$

18. Let A and B be two events such that P (A) = 0.6, P (B) = 0.2, and P (A | B) = 0.5. Then P (A′ | B′) equals

a) $\frac{1}{10}$ b) $\frac{3}{10}$ c) $\frac{3}{8}$ d) $\frac{6}{7}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A) : The function $f : R \to [0, \infty)$ $difined\ by\ f(x) = x^2$ is onto.

Reason (R) : Range of the function $f : R \to [0, \infty)$ $difined\ by\ f(x) = x^2$ is $[0, \infty)$.

20. Assertion (A) : The function$f(x) = \tan x - x$ always increases .

Reason (R) : Derivative of the function $f(x) = \tan x - x$ w.r.t. s is $\sec^2 x - 1$.

Section B
(Each question carries 2 marks)

21. Evaluate $\cos\left[\sin^{-1}\frac{1}{4} + \sec^{-1}\frac{4}{3}\right]$.

OR

Evaluate $\sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

22. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of curve changing when $x = 3$?

23. Prove that the function $f(x) = tanx - 4x$ is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

24. Show that the local maximum value of $x + \frac{1}{x}$ is less than local minimum value.

OR

If at $x = 1$, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2], find the value of a.

25. Evaluate : $\int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}$ dx.

Section C

(Each question carries 3 marks)

26. If $x = \sin t$ and $y = \sin pt$, $prove\ that$ $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

27. Evaluate $\int \frac{1-x^2}{x(1-2x)}\, dx$

28. Evaluate : $\int_0^{\pi} \frac{x\, dx}{1+\sin x}$

OR

Evaluate : $\int_0^{\frac{\pi}{4}} \frac{\sin x.\cos x}{\cos^4 x+\sin^4 x}\, dx$

29. Solve the differential equation $x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0\ and\ x = 1, y = \frac{\pi}{2}$.

OR

Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$; $y = 0\ when\ x = 1$.

30. Solve the following Linear Programming Problem graphically :

Maximise $Z = 4x + y$

subject to the constraints :

$x + y \leq 50,\ 3x + y \leq 90,\ x \geq 0,\ y \geq 0.$

31. Four balls are to be drawn without replacement from a box containing 8 red and 4 white balls. If X denotes the number of red ball drawn, find the probability distribution of X.

Section D

(Each question carries 5 marks)

32. If $R_1 and\ R_2$ are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

OR

Prove that the function $f : [0, \infty) \to [-5, \infty)\ defined\ by\ f(x) = 9x^2 + 6x - 5$ is bijective.

33. If $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations :

$3x - 2y + 3z = 8\ ;\quad 2x + y - z = 1\quad and\quad 4x - 3y + 2z = 4\ .$

34. Make a rough sketch of the region $\{(x, y) : 0 \leq y \leq x^2 + 3,\ 0 \leq y \leq 2x + 3,\ 0 \leq x \leq 3\}$ and find its area, using method of integration.

35. Find the foot of the perpendicular dawn from $(1,2,3)$ on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also find its equation and length of perpendicular.

OR

Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}\ and\ \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. Also find the equation of line of shortest distance.

Section E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First and second case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two Sub-parts of 2 marks each.)

36. **Case- Study-I:-** Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture , suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P_1(6,8,4) , P_2 (21,8,4), P_3 (21,16,10) and P_4 (6,16,10)

Answer the following questions using the above information.

i) What are the components to the two edge vectors defined by $\vec{A}$ = Position Vector of P_2 – Position Vector of P_1 and $\vec{B}$ = Position Vector of P_4 – Position Vector of P_3?

ii) Write the vector in standard notation with $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ (where $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are the unit vectors along the three axes).

iii) What are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$ and in what units?

OR

What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?

37. **Case-Study 2**: Read the following passage and answer the questions given below.

A potter made a mud vessel, where the shape of the pot is based on f(x) = |x – 3| + | x-2|, where f(x) represents the height of the pot.

(i) When x > 4, what will be the height in terms of x ?

(ii) Find $\frac{dy}{dx}$ at x = 3

(iii) When the x value lies between (2,3) then redefine the function in terms of x.

OR

If the potter is trying to make a pot using the function $f(x) = [x]$,will he get a pot or not? Why?

38. **Case-Study 3** : Read the following passage and answer the questions given below.

A car manufacturing factory has two plants, X and Y. Plant X manufactures 70% of cars and plant Y manufactures 30%. 80% of the cars at plant X and 90% of the cars at plant Y are rated of standard quality.

i) If a car is chosen at random, what is the probability that it is of standard quality?

ii) If it is known that the car chosen is of standard quality, what is the probability that it has come from plant X?