DAV PUBLIC SCHOOLS , ODISHA SET-2
PRE-BOARD EXAMINATION
(2023-24)

- Please check that this question paper contains 5 printed pages.
- Check that this question paper contains 38 questions.
- Write down the Serial Number of the question in the left side of the margin before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed 15 minutes prior to the commencement of the examination. The students will read the question paper only and will not write any answer on the answer script during this period.

CLASS- XII

SUB :MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks:80**

General Instructions:

1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is Compulsory. However, there are internal choices in some questions.
2. Section **A** has **18 MCQ's** and **02** Assertion-Reason based questions of **1** mark each.
3. Section **B** has **5Very Short Answer (VSA)**- type questions of **2** marks each.
4. Section **C** has **6 Short Answer (SA)**- type questions of **3** marks each.
5. Section **D** has **4 Long Answer (LA)-** type questions of **5** marks each.
6. Section **E** has **3** source based/case based/passage based/integrated units of assessment (**4** marks each) with sub parts.

SECTION-A

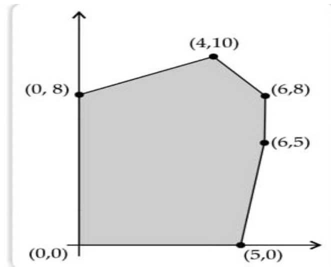
1. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to
   (a) A (b) I -A (c) I +A (d) 3A
2. If A is a square matrix of order 3, with $|A| = 4$, then $|adj(adj(A))|$ is
   (a) 128 (b) 256 (c) 64 (d) 16
3. If A and B are square matrices of order 3 each and $|A| = 2, |B^{-1}| = 3$, then $|2(AB)^{-1}|$ is:
   (a) $\frac{4}{3}$ (b) $\frac{3}{16}$ (c) 12 (d) 16
4. If f(x)= |cos x|, then
   (a) f is everywhere continuous and differentiable
   (b) f is everywhere continuous but not differentiable at $x = n\pi, n \in Z$
   (c) f is everywhere continuous but not differentiable at $x = (2n+1)\frac{\pi}{2}, n \in Z$
   (d) none of the above
5. The direction ratios of the line $x = -3, \frac{y-4}{3} = \frac{2-z}{1}$ are
   (a) $0, 1, -1$ (b) $1, 3, -1$ (c) $0, 3, -1$ (d) $2, 3, -1$
6. The sum of degree and order of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 5$.
   (a) 1 (b) 2 (c) 3 (d) 0
7. Minimize(Z) = 3x + 2y
   Subject to constraints, $x + y \geq 8, 3x + 5y \leq 15, x \geq 0$ and $y \geq 0$ has
   (a) one solution
   (b) no feasible solution
   (c) two solution
   (d) infinitely many solutions

8. If $\vec{a}$ is any vector, then $|\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2$ is
   (a) $|\vec{a}|^2$ (b) $2|\vec{a}|^2$ (c) $3|\vec{a}|^2$ (d) $4|\vec{a}|^2$

9. The value of $\int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x+\cot x} dx$ is equal to
   (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) none

10. The total number of possible matrices of order 2×3 with each entry 2 or 3 is
    (a) 6 (b) 32 (c) 512 (d) 64

11. The feasible solution for a LPP is shown in given figure. Let $Z = 3x - 4y$ be the objective function. Maximum of Z occurs at
    (a) $(5, 0)$
    (b) $(6, 5)$
    (c) $(6, 8)$
    (d) $(4, 10)$



12. The projection of vector $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$ along $\vec{b} = \hat{i}+2\hat{j}+2\hat{k}$ is
    (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) 2 (d) $\sqrt{6}$

13. If for a matrix $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is:
    (a) ± 3 (b) -3 (c) ± 1 (d) 1

14. A and B are two students. Their chances of solving a problem correctly are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If the probability of their making a common error is $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is
    (a) $\frac{1}{12}$ (b) $\frac{1}{40}$ (c) $\frac{10}{13}$ (d) $\frac{10}{11}$

15. Integrating factor of $x\frac{dy}{dx} - y = x^4 - 3x$ is
    (a) x (b) log x (c) 1/x (d) –x

16. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a}\times\vec{b}| = 12$ then the value of $\vec{a}.\vec{b}$ is
    (a) $6\sqrt{3}$ (b) $8\sqrt{3}$ (c) $12\sqrt{3}$ (d) None of these

17. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to:
    (a) $\frac{\cos x}{2y-1}$ (b) $\frac{\cos x}{1-2y}$ (c) $\frac{\sin x}{1-2y}$ (d) $\frac{\sin x}{2y-1}$

18. If direction ratios of a line are $2, 6, -3$ and it makes acute angle with z-axis, then its direction cosines are
    (a) $\frac{2}{7}, \frac{6}{7}, -\frac{3}{7}$ (b) $\frac{2}{7}, -\frac{6}{7}, -\frac{3}{7}$ (c) $-\frac{2}{7}, -\frac{6}{7}, -\frac{3}{7}$ (d) $-\frac{2}{7}, -\frac{6}{7}, \frac{3}{7}$

ASSERTION-REASON BASED QUESTIONS

19. In the following question, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
    (a) Both A and R are true and R is the correct explanation of A
    (b) Both A and R are true but R is not the correct explanation of a.
    (c) A is true but R is false
    (d) A is false but R is true.
    Assertion (A) : $f(x) = -2 + |x-1|$ has minimum value at x = 1.
    Reason (R) : When $\frac{d}{dx}(f(x)) < 0$ for all $x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0$ for all $x \in (a, a+h)$ where 'h' is an infinitesimally small positive quantity, then f(x) has a minimum at x = a. provided f(x) is continuous at x = a.

20. In the following question, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is false but R is true
(d) Both A and R are false.
Assertion (A): Let f : R $\rightarrow$ Rdefined by $f(x) = \frac{|x-1|}{x-1}$ is a bijective function.
Reason (R): A function f : A $\rightarrow$ B is said to be bijective if range of the function is codomain.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Show that the function f : R→R defined by f(x)= [x], x∈ R, where [x] is the greatest integer function is neither one-one nor onto.

OR

Evaluate: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

22. Find $\int \frac{1}{\cos^2 x(1-\tan x)^2}\, dx$

OR

Evaluate $\int_0^1 x(1-x)^n\, dx$

23. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x-coordinate 1, y-coordinate changing twice as fast as x-coordinate. Find the value of a.

24. Find the interval(s) in which the function f(x)= sinx + cosx, x∈ $(0, \frac{\pi}{2})$is strictly increasing or decreasing

25. Prove that $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at $x = \frac{\pi}{6}$.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Find $\int \frac{dx}{(1+e^x)(1-e^{-x})}$

27. If $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$then prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

.

28. Evaluate:$\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$

OR

Evaluate: $\int_0^4 (|x| + |x-2|)\, dx$

.

29. Solve the differential equation $(x^2 - y^2)dx + 2xydy = 0$

OR

Find the general solution of the differential equation $ydx - (x + 2y^2)\, dy = 0.$

30. Solve graphically, the maximum value of Z = 2x + 5y, subject to constraints given below:

$2x + 4y \leq 8$, $3x + y \leq 6$, $x + y \leq 4$, $x \geq 0, y \geq 0$

31. An instructor has question bank consisting of 300 easy true/false questions, 200 difficult true/false questions , 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that, it is a multiple choice question.

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the mean of the distribution.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Find the area of smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$ by using integration

33. Let A = {1 2, 3, ...,9} and R be the relation in A×A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A×A. Prove that R is an equivalence relation.

OR

Consider $f: [0, \infty) \rightarrow [-9, \infty)$given by $f(x) = 5x^2 + 6x - 9$. Prove that f is bijective.

34. Determine the product of $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$and $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, then use to solve the system of equations $x - y + z = 4, x - 2y - 2z = 9$ and $2x + y + 3z = 1$.

35. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection.

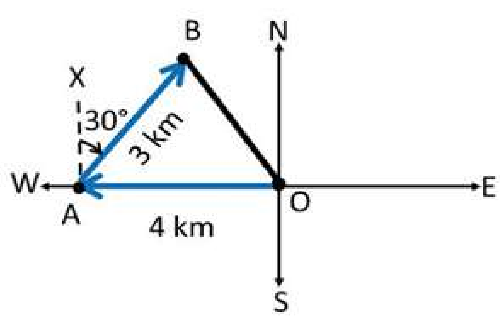
OR

Find the image of the point (1,6,3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

SECTION-E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study question have three sub-parts (i),(ii),(iii) of marks 1,1,2 respectively. The third case study question has two sub-parts of 2 marks each).

36. **Case-Study1:** Nanci walks 4km towards west, then she walk 3km in a direction 30°east of north and stops.



Based on the above information, answer the following questions

(i) Find the position vector of A

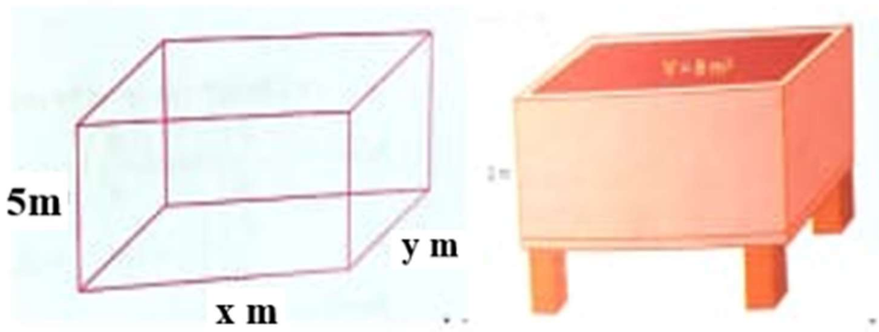
(ii) Find area of triangle OAB.

(iii) Determine Nanci displacement from her initial point of departure.

OR

Find the direction cosines of Nanci displacement from her initial point of departure.

37. **Case-Study 2**: On the request a housing society, a construction agency design a tank with the help of an architect. Tank consists of rectangular base with rectangular sides open at the top, so that its depth is 5m and volume is $20m^3$ as shown below



i) If x and y represents the length and breadth of its rectangular base, then find the relation between the variable.

ii) If construction of tank cost Rs 70 per sq.metre for the base and Rs.45 per square metre for sides, then find cost 'C' in terms of x.

iii) If 'C' is to be minimized find the value of 'x'

OR

If $C = 80 + 80\left(x + \frac{4}{x}\right)$ and we want to minimize the cost 'C' then what will the value of 'x'.

38. **Case-Study 3**: After observing attendance register of Class-XII, Academic committee comes on conclusion that, 30% students have 100% attendance and 70% students are irregular to attend class. It was found that 80% of all students who have 100% attendance secured 95% and above in XII Board exam where 10% irregular students have secured 95% and above marks.

i) At the end of the session, one student is chosen at random from the class has secured 95% and above marks, find the probability that the students has 100% attendance.

ii) Find the total probability of the selected student having 95% and above marks in the class.